Math 5 - Trigonometry Exam 3 Solutions: Chapter 5

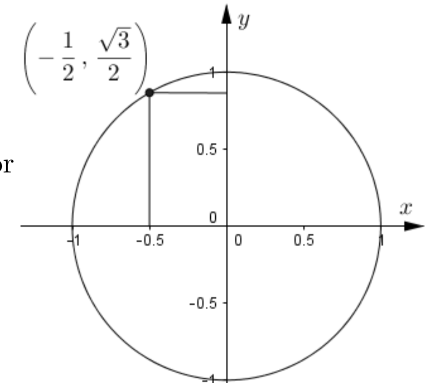
Write all responses on separate paper. Remember to organize your work clearly. You may *not* use your books, notes or a calculator on this exam.

1. If the arclength $t = \dfrac{26\pi}{3}$ is traced counterclockwise along the unit circle from $(1,0)$ then

   (a) What is the reference number $\bar{t}$ for this t ?
   ANS: $t = \dfrac{26\pi}{3} = 8\pi + \dfrac{2\pi}{3}$ so $\bar{t} = \dfrac{\pi}{3}$

   (b) What are the (x, y) coordinates of the terminal point P for this t?
   $P\left(\dfrac{26\pi}{3}\right) = \left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

   (c) Draw the unit circle and plot the terminal point P on it.



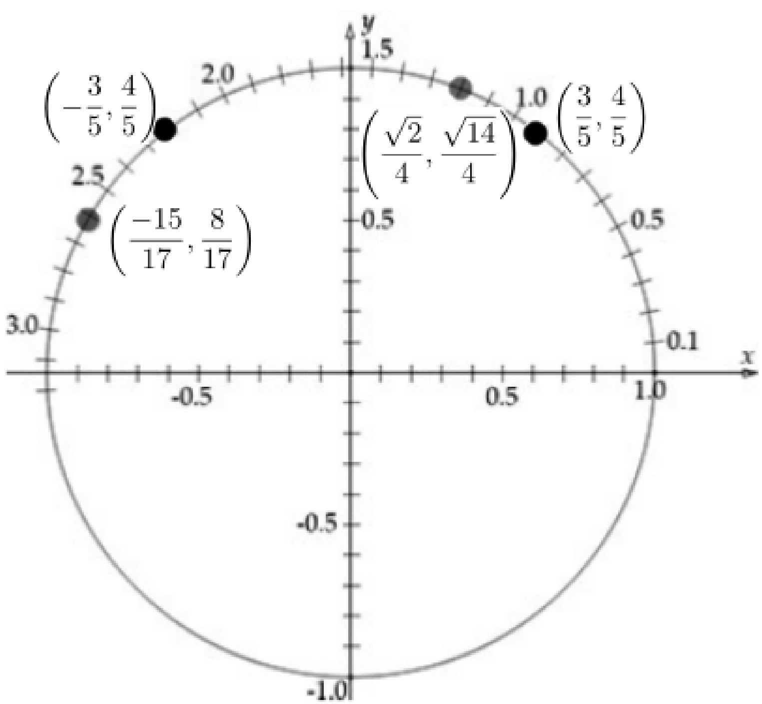
2. Use the unit circle shown at right to answer the following.

   (a) Plot the point $\left(\dfrac{-15}{17}, \dfrac{8}{17}\right) \approx (-0.88, 0.47)$ on the unit circle. Start by doing division to approximate these fractions as decimals to the nearest hundredth.

   (b) Approximate to the nearest tenth the smallest value of t that will lead to the terminal point $P = \left(\dfrac{-15}{17}, \dfrac{8}{17}\right)$
   ANS: $t \approx 2.6$

   (c) Plot the point on the circle in the first quadrant where $x = \dfrac{\sqrt{2}}{4} \approx 0.35$ Find and simplify the exact value of the y-coordinate at that point.
   ANS:$y = \sqrt{1 - \dfrac{2}{16}} = \dfrac{\sqrt{14}}{4}$

   (d) Use the unit circle to approximate to the nearest tenth two values of $t \in [0, \pi]$ where $\sin(t) = \dfrac{4}{5}$
   ANS: $t \approx 0.9$ or $3.14 - 0.9 \approx 2.2$



3. Suppose that $\sin(t) = \dfrac{\sqrt{15}}{4}$ and $\cos(t) < 0$. Simplify the values of
   $\cos(t) = -\sqrt{1 - \dfrac{15}{16}} = -\dfrac{1}{4}$, $\tan(t) = \dfrac{\sin(t)}{\cos(t)} = -\sqrt{15}$, $\sec(t) = -4$, $\csc(t) = \dfrac{4\sqrt{15}}{15}$

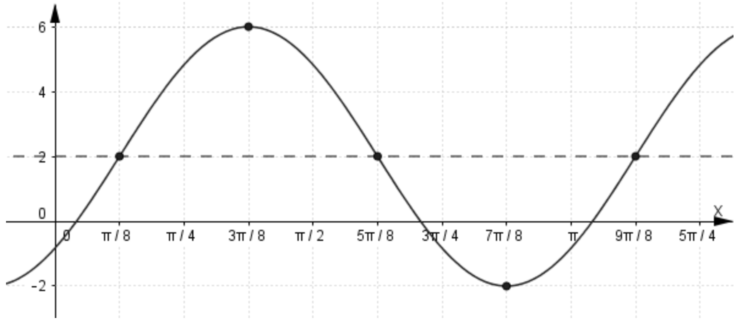
4. (15 points) Consider the the function $f(x) = 2 + 4\sin\left(2x - \frac{\pi}{4}\right)$

   (a) Find amplitude, period, line of equilibrium and the horizontal shift of this sinusoid.
   ANS: Amplitude = 4, period $= \frac{2\pi}{2} = \pi$. The equilibrium line is $y = 2$ and the phase shift is $\frac{\pi}{8}$

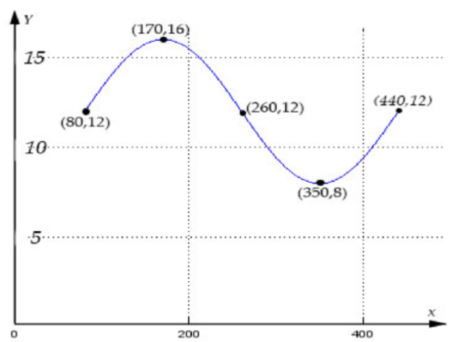
   (b) Construct a table of values showing five points along one oscillation of the wave: points at equilibrium and at maximum displacement from equilibrium.
   ANS:

x	$\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$	$\frac{9\pi}{8}$
y	2	6	2	-2	2

   (c) Carefully construct a large graph,including the points in your table.



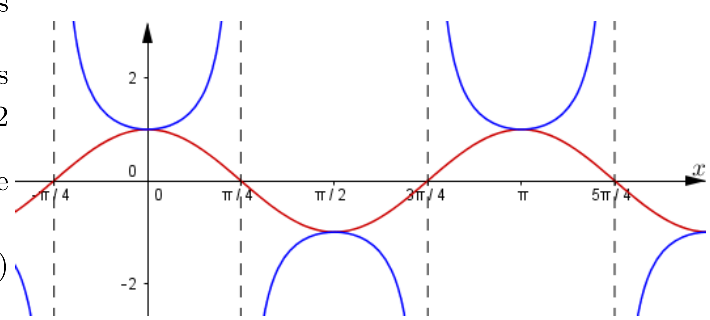
5. Find the amplitude, period, line of equilibrium and horizontal shift and use these to write the equation for the sinusoid whose graph is shown.
   The Amplitude is 4, the period is $\frac{2\pi}{B} = 360 \Leftrightarrow B = \frac{\pi}{180}$, the equilibrium line is $y = 12$ and the horizontal shift is $\phi = 80$ so the equation is $\boxed{y = 12 + 4\sin\left(\frac{\pi}{180}(x - 80)\right)}$



6. Consider the function $y = \sec(2x)$

   (a) Describe the set of all values of x where y is undefined.
   y is undefined where $\cos(2x) = 0$ that is, points where $2x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$. Dividing by 2 gives $x = \frac{\pi}{4} + \frac{k\pi}{2} = \frac{(2k+1)\pi}{4}$, an odd multiple of π over 4.

   (b) Draw graphs of $y = \cos(2x)$ and $y = \sec(2x)$ together in the same (x, y) coordinate plane.



7. Simplify each of the following

   (a) $\sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right) = -\frac{\pi}{6}$

   (b) $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{4}{3}$