Math 5 - Trigonometry **Name (Print):** ____________________

Exam 2: Chapter 2 Solutions

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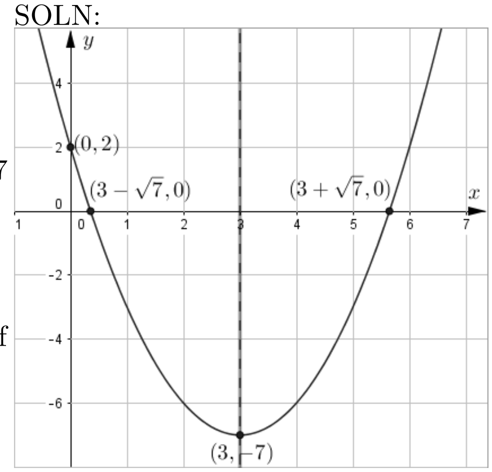
1. Find the domain of each of the following functions.
   (a) $f(x) = \sqrt{7-x}$
   SOLN: Domain $= \{x | 7 - x \geq 0\} = (-\infty, 7]$
   (b) $g(x) = \dfrac{x}{9-x^2}$ SOLN: Domain $= \{x | 9 - x^2 \neq 0\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

2. Compute and simplify the average rate of change of over the given interval. Hint: recall that the average rate of change is of $f(x) = x^2 + 2x + 3$ on the interval $[a, b]$ is the slope of the line connecting $(a, f(a))$ with $(b, f(b))$.
   (a) $[a, b] = [0, h]$ SOLN: $\dfrac{f(h) - f(0)}{h} = \dfrac{h^2 + 2h + 3 - 3}{h} = h + 2$
   (b) $[a, b] = [-h, h]$ SOLN: $\dfrac{f(h) - f(-h)}{h} = \dfrac{h^2 + 2h + 3 - (h^2 - 2h + 3)}{h} = 4$

3. Consider the quadratic function $f(x) = x^2 - 6x + 2$

   SOLN:



   (a) Express the quadratic function in standard (vertex) form: $y = a(x-h)^2 + k$
   SOLN: $f(x) = x^2 - 6x + (9 - 9) + 2 = x^2 - 6x + 9 - 7$ $\boxed{= (x-3)^2 - 7}$ so the vertex is at $(h, k) = (3, -7)$
   (b) Find the coordinates of the $x-$intercepts of the parabola.
   SOLN: $f(x) = 0 \Leftrightarrow (x-3)^2 = 7 \Leftrightarrow x = 3 \pm \sqrt{7}$
   (c) Carefully construct a large graph, showing the coordinates of the vertex and all intercepts.

4. Find the range of the given function and express that in interval notation.
   (a) $f(x) = 10 - 4(x-1)^2$.
   SOLN: $f(x) \leq 10$ so the range is $(-\infty, 10]$
   (b) $f(x) = -2x^2 + 8x + 1$
   SOLN: $f(x) = -2(x-2)^2 + 9 \leq 9$ so the range is $(-\infty, 9]$

5. Consider the quadratic $f(x) = x^2$. What sequence of transformations is required to transform this function to $g(x) = 5 - \frac{1}{2}(x+3)^2$? SOLN:

   - horizontal shift: 3 left, $y = (x+3)^2$
   - vertical shrink: by $\frac{1}{2}$, $y = \frac{1}{2}(x+3)^2$
   - reflection: Across the $x-$axis, $y = -\frac{1}{2}(x+3)^2$
   - vertical shift: up 5, $y = 5 - \frac{1}{2}(x+3)^2$

6. Suppose $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-2}$. Find a formula for and determine the domain of

   (a) $(g \circ f)(x) = \frac{1}{\sqrt{x-2}}$ has a domain of $(2, \infty)$.

   (b) $(f \circ g)(x) = \frac{1}{\sqrt{x}-2}$ has a domain of $[0, 4) \cup (4, \infty)$.

7. Find a formula for the inverse function of $f(x) = \frac{1}{2}x + 1$ and sketch a graph for $y = f(x)$ and $y = f^{-1}(x)$ together showing the symmetry through the line $y = x$. SOLN: Solve $y = \frac{1}{2}x + 1$ for $x = 2(y-1)$ to get the formula for $f^{-1}(x) = 2x - 2$