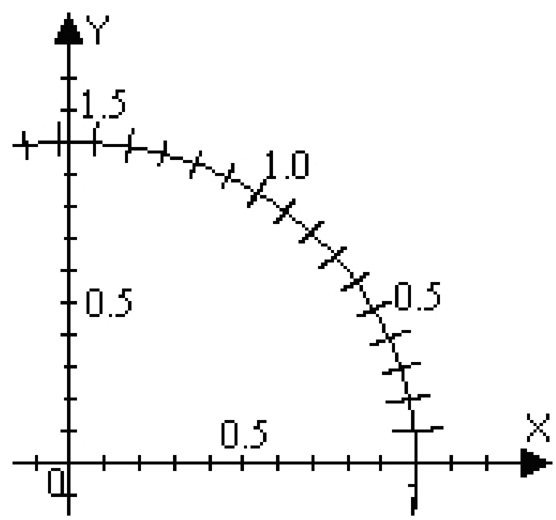
Math 5 – Trigonometry – Chapter 5 Fair Game.

1. If the arclength $t = \dfrac{29\pi}{6}$ is traced counterclockwise along the unit circle from (1,0) then
   a. What is the reference number for t ?
   b. What are the coordinates of the terminal point $P(x,y)$?
   c. Draw the unit circle and plot the terminal point $P(x,y)$.

2. For arclength $t = \dfrac{31\pi}{6}$ extending counterclockwise along the unit circle from (1,0)
   a. Find the reference number for t.
   b. Find the coordinates of the terminal point $P(x,y)$.
   c. Illustrate this point's position on a plot of the unit circle.

3. Consider the point $\left(\dfrac{5}{13}, \dfrac{12}{13}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t) = \dfrac{5}{13} \approx 0.38$
   c. Approximate to the nearest tenth the interval in the first quadrant where $\dfrac{5}{12} \le \tan(t) \le \dfrac{12}{5}$



4. Consider the point $\left(\dfrac{8}{17}, \dfrac{15}{17}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram to approximate to the nearest tenth a value of t so that $\cos(t) = \dfrac{8}{17} \approx 0.47$
   c. Approximate to the nearest tenth a value of t so that $\tan(t) = \dfrac{8}{15}$

5. Recall that a function is even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$. Of the six trigonometric functions: sin(x), cos(x), tan(x), sec(x), csc(x) and cot(x)
   a. Which functions are even?
   b. Which functions are odd?

6. Suppose that $\cos(t) = \dfrac{\sqrt{91}}{10}$ and point and sin(t) < 0.

   Find $\sin(t), \tan(t), \sec(t), \csc(t)$ and $\cot(t)$.

7. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for t is in quadrant III.

8. Find the amplitude, period and phase shift of $y = 5 + 5\sin\left(20\pi\left(x - \dfrac{1}{50}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

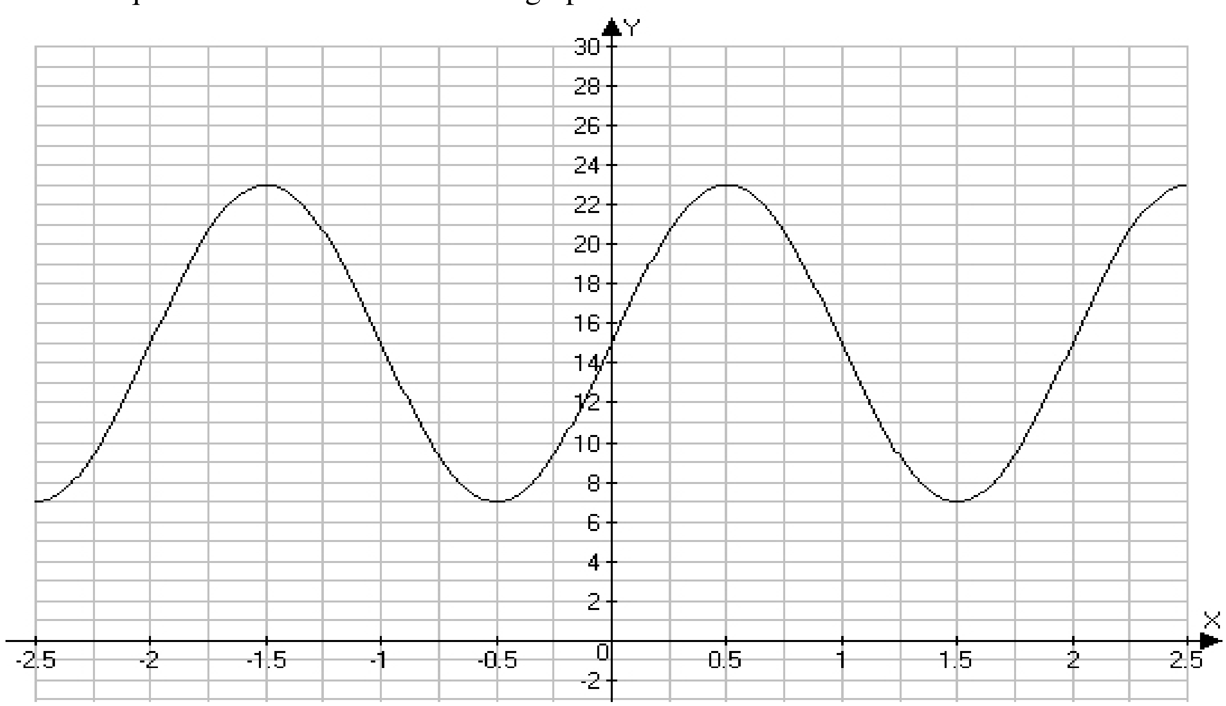
9. Suppose that $\cos(t) = \dfrac{99}{101}$ and point and sin(t) < 0.

   Find $\sin(t), \tan(t), \sec(t), \csc(t)$ and $\cot(t)$.

10. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for t is in quadrant III.

11. Find the amplitude, period and phase shift of $y = 5 + 4\sin\left(2\pi\left(x + \dfrac{1}{4}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

12. Find an equation for the sinusoid whose graph is shown:



13. Consider the function $f(x) = \tan\left(\frac{\pi}{2}x\right)$.

a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

14. Suppose cos t = 9/28 and t is in the first quadrant. Find the following:

a. $\cos(t+\pi)$
b. $\cos\left(t+\frac{\pi}{2}\right)$
c. $\cos\left(\frac{\pi}{2}-t\right)$

15. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x-\frac{1}{2}\right)\right)$.

a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

16. Suppose *sin t* = 16/65 and t is in the first quadrant. Find the following:

a. $\sin(t+\pi)$
b. $\sin\left(t+\frac{\pi}{2}\right)$
c. $\sin\left(\frac{\pi}{2}-t\right)$

17. Complete the table of values for $f(t) = \cos(\pi t) + 2\sin(\pi t)$, plot the points and sketch a graph.

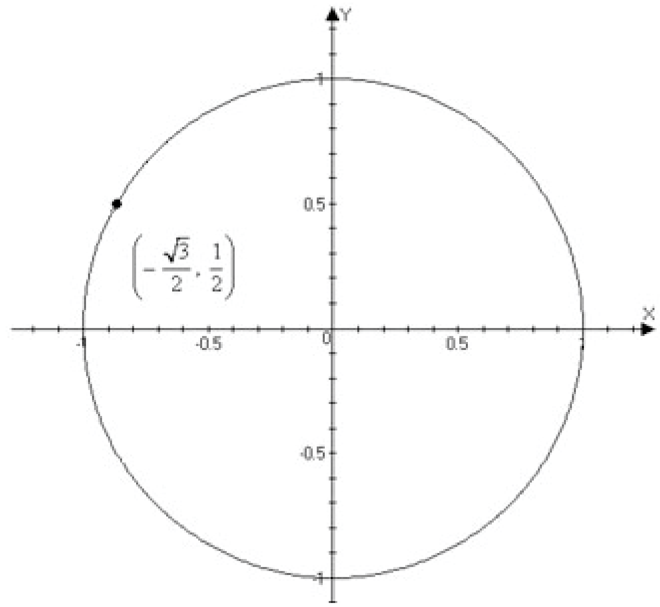
t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$									
$2\sin(\pi t)$									
$f(t)$									

18. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives the height of a rider t minutes after boarding the Millennium Wheel.

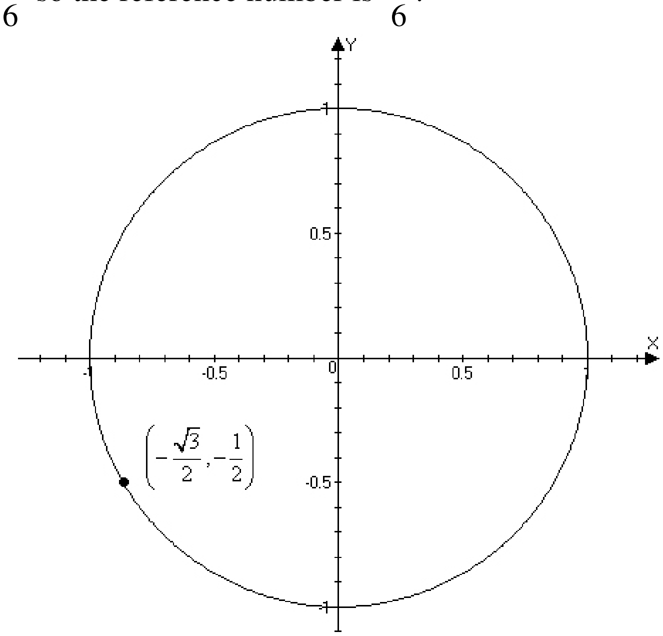
19.

Math 5 – Trigonometry – Chapter 5 Fair Game Solutions

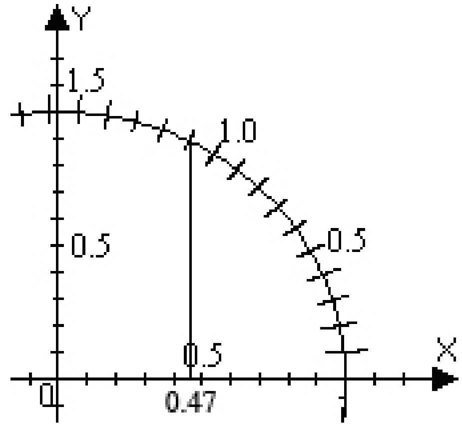
1. For arclength $t = \frac{29\pi}{6}$ extending counterclockwise along the unit circle from (1,0)
   a. Find the reference number for *t*.
      ANS: $t = \frac{(12+12+5)\pi}{6} = 2\pi + 2\pi + \frac{5\pi}{6}$ so the reference number is $\frac{5\pi}{6}$.
   b. Find the coordinates of the terminal point *P*(*x*,*y*).
      ANS: Since this point is in the second quadrant, $x < 0$ and $y > 0$ so
      $x = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $y = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
   c. Illustrate this point's position on a plot of the unit circle.
      ANS: The point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.



2. For arclength $t = \frac{31\pi}{6}$ extending counterclockwise along the unit circle from (1,0)
   a. Find the reference number for *t*.
      ANS: $t = \frac{31\pi}{6} = \frac{(12+12+6+1)\pi}{6} = 2\pi + 2\pi + \pi + \frac{\pi}{6}$ so the reference number is $\frac{\pi}{6}$.
   b. Find the coordinates of the terminal point *P*(*x*,*y*).
      ANS: Since this point is in the third quadrant, both *x* and *y* are negative and so $x = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ and
      $y = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.
   c. Illustrate this point's position on a plot of the unit circle.
      ANS: The point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$



3. Consider the point $\left(\frac{8}{17},\frac{15}{17}\right)$



   a. Verify that the point lies on the unit circle.

   ANS: $\left(\frac{8}{17}\right)^2+\left(\frac{15}{17}\right)^2=\frac{64}{289}+\frac{225}{289}=\frac{289}{289}=1$

   b. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t)=\frac{8}{17}\approx 0.47$

   ANS: A vertical segment is drawn from 0.47 on the *x*-axis intersects the circle at *t* near 1.1

   c. Approximate to the nearest tenth a value of *t* so that $\tan(t)=\frac{8}{15}$

   ANS: Since cot(*t*) = cos(*t*)/sin(*t*) = 8/15 and tan(π/2 – *t*) = cot(*t*). So choose *t* = 1.6 – 1.1 = 0.5

4. Suppose that $\cos(t)=\frac{99}{101}$ and point and sin(*t*) < 0.

   Find $\sin(t),\tan(t),\sec(t),\csc(t)$ and $\cot(t)$.

   ANS: $\sin(t)=-\sqrt{1-\cos^2 t}=-\sqrt{1-\left(\frac{99}{101}\right)^2}=-\sqrt{1-\frac{9801}{10201}}=-\sqrt{\frac{10201-9801}{10201}}=-\sqrt{\frac{400}{10201}}=-\frac{20}{101}$

   Thus $\tan(t)=-\frac{20}{99}$; $\sec(t)=\frac{101}{99}$; $\csc(t)=-\frac{101}{20}$; $\cot(t)=-\frac{99}{20}$

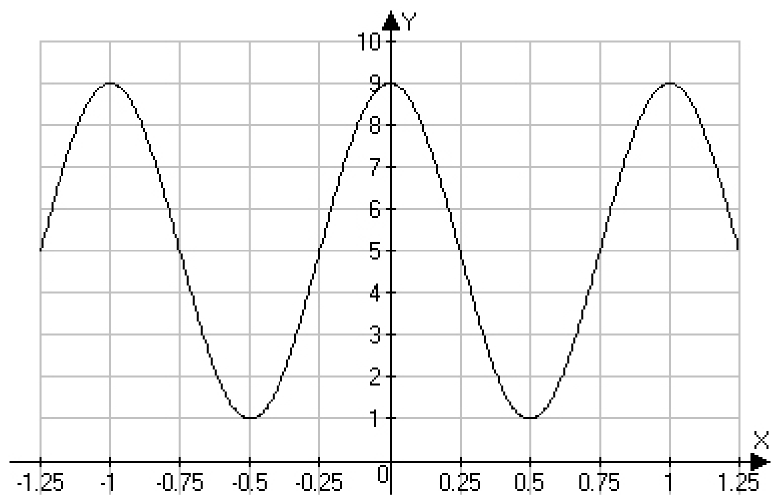
5. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for *t* is in quadrant III.

   ANS: Starting with $\cos^2 t+\sin^2 t=1$, divide through by $\cos^2 t$ to obtain $1+\tan^2 t=\sec^2 t$. Since sec(*t*) is negative in quadrant III, $\sec t=-\sqrt{1+\tan^2 t}$

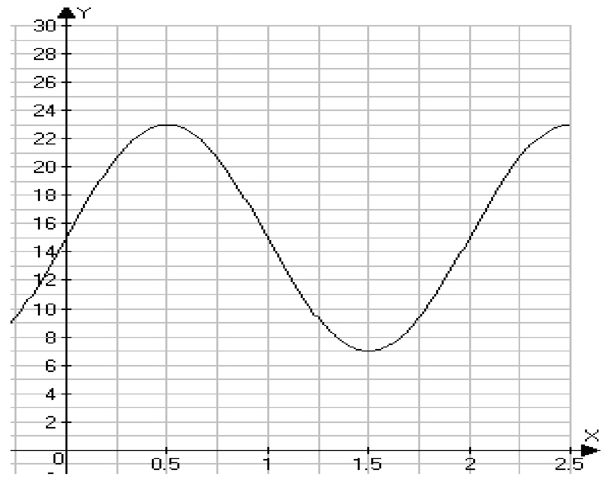
6. Find the amplitude, period and phase shift of $y=5+4\sin\left(2\pi\left(x+\frac{1}{4}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

   ANS: The amplitude is 4,
   the period is 1
   and the phase angle is –1/4.

   Graph is shown at right.



7. Find an equation for the sinusoid whose graph is shown:



ANS: The lowest point is at y=7 and the highest point is at 23 so the line of equilibrium is at the average of these: y = (7+23)/2 = 15. and the amplitude is (23 – 7)/2 = 8.

The two peaks shown in the graph here are where x = 0.5 and x = 2.5, so the period is 2.5 – 0.5 = 2.
Thus an equation for the sinusoid is $y = 15 + 8\sin(\pi x)$.

8. Consider the function $f(x) = \tan\left(\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right)$.

a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.

ANS: We want the input to the tangent to be $\pm\frac{\pi}{2}$, that is

$$\frac{\pi}{2}\left(x - \frac{1}{2}\right) = \pm\frac{\pi}{2} \Leftrightarrow x - \frac{1}{2} = \pm 1 \Leftrightarrow \boxed{x = \frac{1}{2} \pm 1 = -\frac{1}{2} \text{ or } \frac{3}{2}}$$

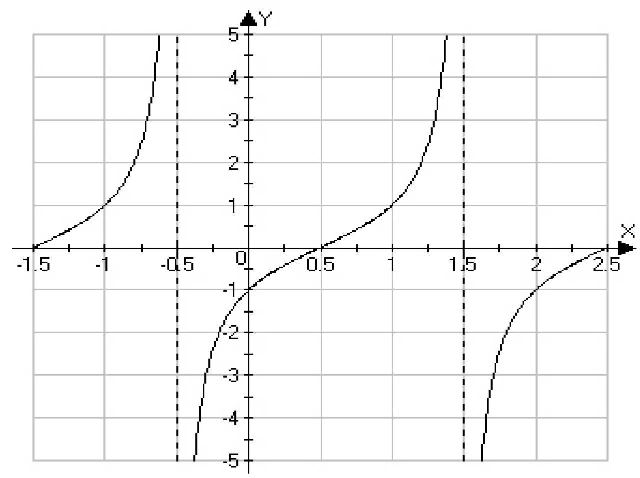
b. Find x-coords where $y = 0$ and $y = \pm 1$.

ANS: We want to find where the input to the tangent function is equal to $\pm\frac{\pi}{4}$, that is

$$\frac{\pi}{2}\left(x - \frac{1}{2}\right) = \pm\frac{\pi}{4} \Leftrightarrow x - \frac{1}{2} = \pm\frac{1}{2}.$$

$$\Leftrightarrow \boxed{x = 0 \text{ or } x = 1}$$

c. Graph of the function showing how it passes through the points where y = -1, y = 0, y = 1 and how it approaches the vertical asymptotes.

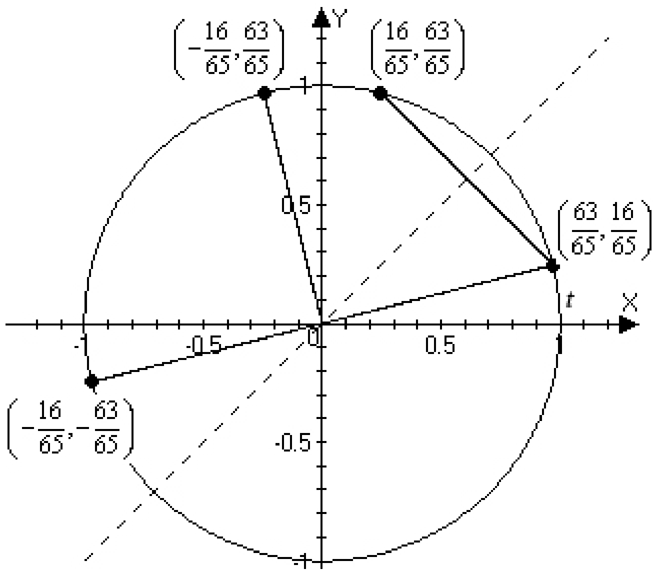


9. Suppose $\sin t$ = 16/65 and t is in the first quadrant. Find the following:

a. $\sin(t + \pi) = -\frac{16}{65}$

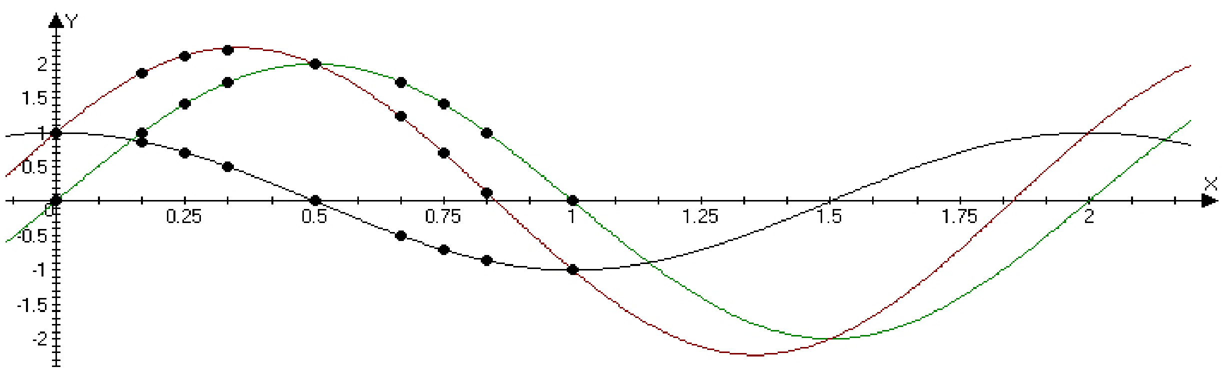
b. $\sin\left(t + \frac{\pi}{2}\right) = -\cos(t) = -\sqrt{1 - \left(\frac{16}{65}\right)^2} = -\sqrt{1 - \frac{256}{4225}} = -\sqrt{\frac{3969}{4225}} = \frac{63}{65}$

c. $\sin\left(\frac{\pi}{2} - t\right) = \frac{63}{65}$



10. Complete the table of values for $f(t)=\cos(\pi t)+2\sin(\pi t)$, plot the points and sketch a graph.

t	0	1/6	1/4	1/3	1/2	2/3	3/4	5/6	1
$\cos(\pi t)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$2\sin(\pi t)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0
$f(t)$	1	$1+\frac{\sqrt{3}}{2}$	$\frac{3\sqrt{2}}{2}$	$\frac{1}{2}+\sqrt{3}$	2	$-\frac{1}{2}+\sqrt{3}$	$\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{3}}{2}$	-1



11. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives the height of a rider *t* minutes after boarding the Millennium Wheel.
ANS: $h(t)=70-65\cos(\pi t/15)$